Modelling Supported Driving as an Optimal Control Cycle: Framework and Model Characteristics

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Outline

Introduction

Control framework formulation

Example controllers

Controller characteristics

Conclusion and future research
Background

- Global research interests in Advanced Driver Assistance Systems and Cooperative Systems
- Increase of Adaptive Cruise Control (ACC) systems equipped vehicles on road
- ACC system automates the longitudinal driving tasks:
  - Cruising mode: maintain free speed
  - Following mode: maintain desired gap
- Induced changes in individual vehicular behaviour and collective traffic flow operations
Existing ACC controllers (or models)

- Widely-used Constant Time Gap policy, i.e. Helly car-following model without time delay
- Overruled by drivers at highly non-stationary conditions and congestions
- Difficult to incorporate cooperative driving concept with Vehicle-Vehicle (V2V) communications
- Unable to fulfil multiple objectives, e.g. maximising safety, efficiency and sustainability
This contribution

- An optimal control framework for driver assistance and cooperative systems
- An ACC controller with human-like behaviour
- A Cooperative ACC (C-ACC) controller that captures vehicle-vehicle collaboration
- New insights into stability characteristics of the controllers
Control assumptions

- Controlled acceleration, i.e. automatic control of throttle and brake pedal
- Information of other vehicles influencing control decisions available
- No delay in the control loop
- Deterministic case
Rolling horizon approach
Cost minimisation in one control cycle

\[ u^* = \arg \min J(x, u|x_0) \]

\[ J(x, u|x_0) = \int_{t_0}^{\infty} e^{-\eta \tau} \mathcal{L}(x, u) d\tau \]

s.t.

\[ \dot{x} = f(x, u), \quad x_0 = x(t_0) \]

- \( x \): system state; \( \mathcal{L} \): running cost
- \( \eta > 0 \): discount factor, cost discounted in the (uncertain) future and decreases exponentially after a horizon \( 1/\eta \)
- \( u^* \): optimal acceleration, can be found by Dynamic Programming approach
Controller design procedure under the framework

1. Define system state and determine state prediction model (i.e. system dynamics equation)
2. Specify cost function under control objectives
3. Find optimal acceleration
4. Verify the controller performance
Example 1: ACC controller

System state: \( \mathbf{x} = (s_n, \Delta v_n, v_n)' \)

State prediction model: system dynamics equation

\[
\dot{\mathbf{x}} = \left( \Delta v_n, u_{n-1} - u_n, u_n \right)'
\]
ACC control objectives

- Maximise safety in following mode, by penalising approaching leader
- Maximise travel efficiency, by penalising deviation from desired speed or free speed
- Maximise driving comfort, by penalising large acceleration/deceleration

\[ \mathcal{L} = \begin{cases} 
  c_1 e^{s_0} \Delta v^2 \cdot \Theta(\Delta v) + c_2 (v_d(s) - v)^2 + \frac{1}{2} u^2 & \text{following} \\
  c_3 (v_0 - v)^2 + \frac{1}{2} u^2 & \text{cruising} 
\end{cases} \]
Optimal acceleration of ACC vehicle

\[
\mathbf{u}_\text{ACC}^* = \begin{cases} 
\frac{2c_1 e^{s_0 s}}{\eta} \left( \Delta v - \frac{s_0 \Delta v^2}{\eta s^2} \right) \cdot \Theta(\Delta v) + \frac{2c_2}{\eta} \left( 1 + \frac{2}{\eta t_d} \right) (v_d(s) - v) & \text{following} \\
\text{decelerate when approaching} & \text{match desired speed} \\
\frac{2c_3}{\eta} (v_0 - v) & \text{cruising} \\
\text{match free speed} & 
\end{cases}
\]

Desired speed \( v_d \) as a function of gap:

Satisfies necessary conditions for plausible car-following models!
Example 2: Cooperative ACC (C-ACC)

- Two CACC follower exchange information (gap and relative speed) via V2V communication
- Negotiate and coordinate their (car-following) behaviour under a common objective
- System state $\mathbf{x} = (s_n, \Delta v_n, v_n, s_{n+1}, \Delta v_{n+1}, v_{n+1})'$
- State prediction model:
  $\dot{\mathbf{x}} = (\Delta v_n, u_{n-1} - u_n, u_n, \Delta v_{n+1}, u_n - u_{n+1}, u_{n+1})'$
C-ACC cost and acceleration

- Control objective: maximising safety, efficiency and driving comfort for both followers
- Joint cost function: sum of costs for two followers

\[ u^*_C-\text{ACC} = \underbrace{u^*_{\text{ACC}}}_{\text{ACC acceleration}} - \frac{2c_1 e^{s_{0}}}{{\eta}} \left( \Delta v_{n+1} - \frac{s_0 \Delta v_{n+1}^2}{2\eta s_{n+1}^2} \right) \cdot \Theta(\Delta v_{n+1}) \]

accelerate when \( \Delta v_{n+1} < 0 \)

\[ -\frac{2c_2}{{\eta^2} t_d} (v_d(s_{n+1}) - v_{n+1}) \]

decelerate when \( v_{n+1} < v_d(s_{n+1}) \)
Anlytical framework for stability analysis

- Consider a generalised acceleration function $u(s, \Delta v, v, s_b, \Delta v_b, v_b)$, with $s_b, \Delta v_b, v_b$ denoting the situation behind

- Find equilibrium gap-speed relation $v_e(s_e)$ by setting $u = 0$ and $\Delta v = 0$

- Insert small disturbances of gap and speed to a vehicle at equilibrium

- Take derivatives of disturbance and get disturbance dynamical equation

- Solve the dynamical equation and find the signs of the roots using Fourier analysis
String stability criteria

ACC controller:

\[ v'_e(s_e)^2 \leq v'_e(s_e)u_{\Delta v} + \frac{u_s}{2} \]

C-ACC controller:

\[ v'_e(s_e)^2 \leq v'_e(s_e)u_{\Delta v} + \frac{u_s}{2} + v'_e(s_e)(u_{\Delta v_b} - u_{v_b}) - \frac{u_{s_b}}{2} \]

stabilisation effects of C-ACC
Neutral string stability line

String stability of ACC platoon is enhanced with:

- larger safety cost weight, more anticipative driving style
- larger time gap, larger following distance
- smaller discount factor, longer prediction horizon
**Convective and absolute instability**

If string instability prevails, in which direction do the disturbances propagate in the $x$-$t$ plane: upstream, downstream or both?

![Graphs showing convective and absolute instability](image)

**Figure:** (a) Convective upstream instability; (b) Absolute instability (Treiber and Kesting, 2013).
Stability diagram using Fourier transform

S: Stability; U: convective Upstream instability; A: Absolute instability; D: convective Downstream instability

![Stability Diagram](attachment:image.png)

Figure: (a) ACC controller; (b) C-ACC controller.
Summary

- An optimal control framework for driver assistance and cooperative systems
- An ACC controller with plausible car-following behaviour
- A C-ACC controller under collaborative driving concept
- Rigorous stability criteria for ACC and C-ACC can be used as guidance for controller design and tuning
- The C-ACC controller produces significantly different string instability property compared to the ACC controller
Future research

- Including delay and inaccuracy in the framework
- Design cooperative vehicle controller to improve stability
- Challenge to model-based traffic state estimation, prediction and control methods in Cooperative Systems
Further reading